CERTIFIED PUBLIC ACCOUNTANT

FOUNDATION LEVEL 1 EXAMINATION

F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE METHODS

DATE: THURSDAY, 31 MARCH 2022

MARKING GUIDE AND MODEL ANSWER

QUESTION ONE

Marking guide	Marks
a) For the construction of a network (1 Mark for each correct rule, maximum 2)	**2**
b) i) Computation of the time estimate (check column 3, 5 and 6 and 1 mark each correct column) from the table, 1 mark for the correct network and 1 mark for the project time	5
ii) Computation of variance and standard deviation (1 mark for correct column 5,6 and 7), 1 mark for correct variance and 1 mark for correct standard deviation	5
Maximum	**10**
c) i) 2 marks for correct computation	2
ii) 2 marks for correct computation	2
iii) 0.5 for each correct critical path	1
iv) 1 Mark for completion time from 25 to 16 weeks, 1 Mark for additional cost of reduction and 1 Mark for completion time	3
Maximum	**8**
Total Marks	**20**

Model answers

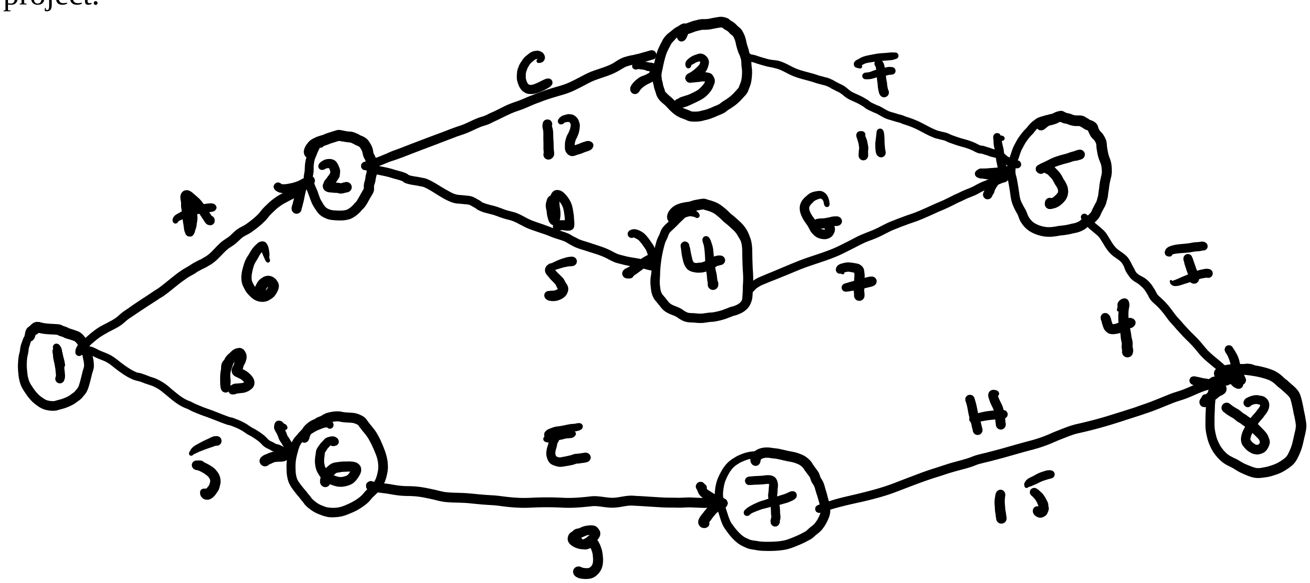
(a) For the construction of a network, we make use of the following rules:

- Each activity is represented by one and only one arrow. This implies that no single activity can be represented twice in the network.
- Not two activities can be identified by the same end events. This implies that there must be no loops in the network.
- Time flows from left to right. Arrows pointing in opposite direction must be avoided.
- Arrows should be kept straight and not curved or bent.
- Avoid arrows which cross each other.
- Use dummies freely in rough draft but final network should not have any redundant dummies.
- Every node must have at least one activity preceding it and at least one activity following it except for the very beginning and at the very end of the network. The
- beginning node has no activities before it and the ending node has no activities following it.
- Only one activity may connect any two nodes. This rule is necessary so that an activity can be specified by giving the numbers of its beginning and ending nodes.

(b)(i) From the three-time estimates, we compute the following table:

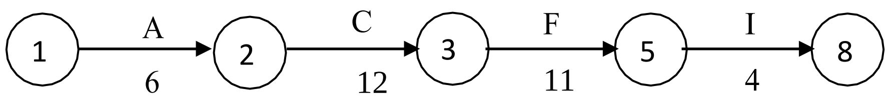
Activity	Optimistic time Estimate (to)	4 x Most likely time estimate	Pessimistic time estimate (tp)	to+ 4tm + tp	Time estimate $t_{e}=\frac{t_0 + 4t_m + t_p}{6}$
1-2	3	24	9	36	6
1-6	2	20	8	30	5
2-3	6	48	18	72	12
2-4	4	20	6	30	5
3-5	8	44	14	66	11
4-5	3	28	11	42	7
6-7	3	36	15	54	9
5-8	2	16	6	24	4
7-8	8	64	18	90	15

With the single time estimates of the activities, we get the following network diagram for the project.



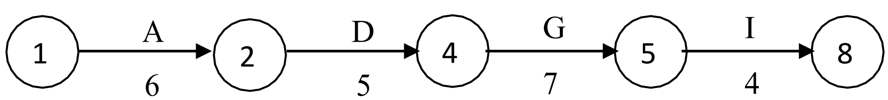
Consider the paths, beginning with the start node and stopping with theend node. There are three such paths for the given project. They are as follows:

Path I



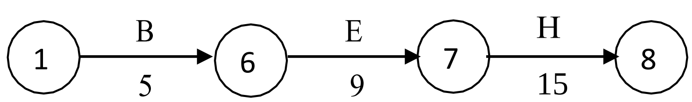
Time for the path: 6+12+11+4 = 33 weeks.

Path II



Time for the path: 6+5+7+ 4= 22 weeks.

Path III



Time for the path: 5+9+15 = 29 weeks. Compare the times for the three paths.
Maximum of {33, 22, 29} = 33. It is noticed that Path I has the maximum time.

Therefore, the critical path is path I: 1-2-3-5-8. The critical activities are A, C, F and I.
The non-critical activities are B, D, G and H.

Project time = 33 weeks.

(ii) Calculation of Standard Deviation and Variance for the Critical Activities:

Critical Activity	Optimistic time estimate (to)	Most likely time estimate (tm)	Pessimistic time estimate (tp)	Range (tp - to)	Standard deviation $\sigma = \frac{t_p - t_0}{6}$	Variance σ^2
A: 1 2	3	6	9	6	1	1

C: 2 3	6	12	18	12	2	4
F: 3 5	8	11	14	6	1	1
I: 5 8	2	4	6	4	2/3	4/9

Variance of project time (Also called Variance of project length) = Sum of the variances for the critical activities = 1+4+1+ 4/9 = 58/9 Weeks.

Standard deviation of project time = $\sqrt{\text{variance}} = \frac{\sqrt{58}}{9} = 2.54$ weeks

c)

Activity	Time(weeks)		Saved	Cost (FRW)			
	Normal	Crash		Normal	Crash	Increase	Per week
A	3	1	2	100,000	300,000	200,000	100,000
B	4	3	1	400,000	600,000	200,000	200,000
C	2	2	0	200,000	200,000	-	-
D	6	4	2	300,000	600,000	300,000	150,000
E	5	4	1	250,000	380,000	130,000	130,000
F	3	2	1	150,000	300,000	150,000	150,000
G	7	4	3	450,000	810,000	360,000	20,000
H	5	4	1	300,000	360,000	60,000	60,000
I	8	5	3	800,000	1,280,000	480,000	160,000
				2,950,000	4,830,000	1,880,000	

(i) The total cost of performing the project in the normal time of 25 weeks is obtained by summing the normal costs for each of the activities. By drawing the network diagram, the critical path is
C, E, G, A, I corresponding to 2+5+7+3+8= 25 weeks.
The total cost = 200,000 + 250,000 + 450,000 + 100,000 + 800,000 = FRW 1,950,000

(ii) Cost per week $= \frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}} = \frac{\text{Cost increase}}{\text{time saved}}$

(iii) If the normal times are replaced with the crashed times, then using the criterion of crashing from the least expensive to most expensive, it is easy to show that there are two critical paths based on crashed times:

- C-E-G-A-I
- C-E-H-A-I

(iv) The saving that the consultancy realizes by reducing the project completion time from 25 to 16 weeks is 9 x 150,000 = FRW 1,350,000.
But the additional cost of reduction is FRW 4,830,000 – FRW 2,950,000 = FRW1,880,000. Hence, it is not economical to reduce all of the activities to their crashed times. Crashing all activities means that both critical and noncritical activities are crashed (expedited). However, there is clearly no economy in

expediting noncritical activities. Hence, it is likely that there is a solution with a project completion time of 16 weeks that costs less than FRW 4,830,000.

QUESTION TWO

Marking Guide	Marks
(a) 1 Mark for each component	**4**
b) 2 Marks for each correct column 4,5 and 6 and 2 Marks for the interpretaion	**8**
c) i) 1 Mark for the definition and 2 Marks for two correct limitations	3
ii) (I) 1 Mark for the formula and 2 Marks for the correct column 3	3
(II) 2 Marks for the correct column 4	2
Maximum	**8**
Total Marks	20

Model Answers

(a) The four components of time series:

Secular Trend:

Secular Trend is also called long term trend or simply trend. The *secular trend* refers to the general tendency of data to grow or decline over a long period of time.

Seasonal Variations:

Seasonal variations occur in the time series due to the rhythmic forces which occurs in a regular and a periodic manner with in a period of less than one year. Seasonal variations occur during a period of one year and have the same pattern year after year. Here the period of time may be monthly, weekly or hourly. But if the figure is given in yearly terms, then seasonal fluctuations does not exist.

Cyclical Variations or Oscillatory Variation:

This is a short-term variation occurs for a period of more than one year. The rhythmic movements in a time series with a period of oscillation (repeated again and again in same manner) more than one year is called a cyclical variation and the period is called a cycle. The time series related to business and economics show some kind of cyclical variations.

Irregular Variation:

It is also called Erratic, Accidental or Random Variations. The three variations trend, seasonal and cyclical variations are called as regular variations, but almost all the time series including the regular variation contain another variation called as random variation. This type of fluctuations occurs in random way or irregular ways which are unforeseen, unpredictable and due to some irregular circumstances, which are beyond the control of human being such as earth quakes, wars, floods, famines, lockouts, etc. These factors affect the time series in the irregular ways. These irregular variations are not so significant like other fluctuations.

(b) Actual results fluctuate up and down according to the day of the week and so moving average of five will be used. The difference between the actual result on any one day (Y) and the trend figure for that day (T) will be the seasonal variation (S) for the day. The seasonal variation for the 15 days is as follows:

Week	**Day**	**Actual (Y)**	**Moving total of five days' output**	**Trend (T)**	**Seasonal variation (Y-T)**
	Monday	80			
	Tuesday	104			
Week 1	Wednesday	94	460	92	2
	Thursday	120	462	92.4	27.6
	Friday	62	468	93.6	-31.6
	Monday	82	471	94.2	-12.2
	Tuesday	110	476	95.2	14.8
Week 2	Wednesday	97	478	95.6	1.4
	Thursday	125	480	96	29
	Friday	64	486	97.2	-33.2
	Monday	84	489	97.8	-13.8
	Tuesday	116	494	98.8	17.2
Week 3	Wednesday	100	496	99.2	0.8
	Thursday	130			
	Friday	66			

You will notice that the variation between the actual results on any one particular day and the trend line average is not the same from week to week. This is because Y-T contains not only seasonal variations but random variations, and an average of these variations can be taken.

c) (i) Simple exponential smoothing (usually referred to as exponential smoothing) is a time series forecasting method that smoothes out random fluctuations of data. It is best used for short-term forecasts in the absence of seasonal or cyclical variations. Similarly, the method does not work very well if the series has a trend. Exponential smoothing weights past data with weights that decrease exponentially with time, thus adjusting for previous inaccuracies in forecasts.

(ii) Consider $\alpha = 0.3$ and the using the smooth model
$F_{t+1} = \alpha Y_t + (1 - \alpha) F_t$ and the errors $e_t = Y_t - F\_t$
Where:

- F_{t+1} is the forecast value of the dependent variable for period $t + 1$
- F_t is the forecast value of the dependent variable for period t
- Y_t is the actual value of the dependent variable for period t

- α is the value of the smoothing constant

The results are shown in the following table

Week	Sales (Y_t)	Forecast (F_t)	Forecast error (e_t)
1	130	130.00	0
2	70	130.00	-60
3	140	112.00	28
4	150	120.40	29.6
5	90	129.28	-39.28
6	180	117.50	62.5
7		136.25	

QUESTION THREE

Marking guide	**Marks**
a)	
1 Mark for each correct column 6,7,8 and 9 (4 columns)	4
Laspeyres method (1 Mark for formula and 1 Mark for substitution and computation)	2
Paasche method (1 Mark for formula and 1 Mark for substitution and computation)	2
Fisher's ideal index (1 Mark for formula and 1 Mark for substitution and computation)	2
Maximum	**10**
b) Steps 1 Mark for each of nine steps	9
Final computation of total cost assignment	1
Maximum	**10**
Total Marks	**20**

Model Answers

(a) To be able to provide the required answers, we fill the following table

	P0"	**Q0**	**Pt**	**Qt**	**P0Q0**	**PtQ0**	**P0Qt**	**PtQt**
	"000"		**"000"**		**"000"**	**"000"**	**"000"**	**"000"**
Wine	9.3	100	4.5	90	930	450	837	405
Beer	6.4	11	3.7	10	70.4	40.7	64	37
Soft drinks	5.1	5	2.7	3	25.5	13.5	15.3	8.1
Total					**1025.9**	**504.2**	**916.3**	**450.1**

The weighted price index for 2014 using Laspeyres method $P.I = \frac{\sum p_t q_0}{\sum p_0 q_0}(100) = \frac{504{,}2}{1025{,}9} \times 100 =$ 49.15

The Paasche method index for 2014

$$P.I = \frac{\sum p_t q_t}{\sum p_0 q_0}(100) = \frac{\$450.1}{\$916.3}(100) = 49.12$$

Fisher's ideal index is

$$\text{Fisher's ideal index} = \sqrt{(\text{Laspeyres'index})(\text{Paasche'sindex})}$$
$$= \sqrt{(49.15)(49.12)} = 49.134$$

(b)As per the Hungarian Method

Step 1: The cost Table

		Jobs			
		1	**2**	**3**	**4**
	A	20	25	22	28
Persons	**B**	15	18	23	17
	C	19	17	21	24
	D	25	23	24	24

Step 2 and Step 3: In each row of the table find out the smallest cost element, subtract this smallest cost element from each element in that row. So, that there will be at least one zero in each row of the new table. This new table is known as First Reduced Cost Table. In each column of the table find out the smallest cost element, subtract this smallest cost element from each element in that column. As a result of this, each row and column has at least one zero element. This new table is known as Second Reduced Cost Table

Step 2

		Jobs			
		1	2	3	4
	A	0	5	2	8
	B	0	3	8	2
Persons	C	2	0	4	7
	D	2	0	1	1

Step 3: Find the Second Reduced Cost Table

		Jobs			
		1	2	3	4
	A	0	5	1	7
	B	0	3	7	1
Persons	C	2	0	3	6
	D	2	0	0	0

Step 4: Determine an Assignment

By examine row A of the table in Step 3, we find that it has only one zero (cell A1) box this zero and cross out all other zeros in the boxed column. In this way we can eliminate cell B1.

Now examine row C, we find that it has one zero (cell C2) box this zero and cross out (eliminate) the zeros in the boxed column. This is how cell D2 gets eliminated.

There is one zero in the column 3. Therefore, cell D3 gets boxed and this enables us to eliminate cell D4.

Therefore, we can box (assign) or cross out (eliminate) all zeros.

The resultant table is shown below:

		Jobs			
		1	**2**	**3**	**4**
Persons	**A**	0	5	1	7
	B	0	3	7	1
	C	2	0	3	6
	D	2	0	0	0

Step 5: The solution obtained in Step 4 is not optimal. Because we were able to make three assignments when four were required.

Step 6:
Cover all the zeros of the table shown in the Step 4 with three lines (since already we made three assignments).

Check row B since it has no assignment. Note that row B has a zero in column 1, therefore check column1. Then we check row A since it has a zero in column 1. Note that no other rows and columns are checked. Now we may draw three lines through unchecked rows (row C and D) and the checked column (column 1). This is shown in the table given below:

		Jobs			
		1	2	3	4
Persons	A	[0]	5	1	7
	B	0	3	7	1
	C	2	0	3	6
	D	2	0	0	0

Step 7:

Develop the new revised table.

Examine those elements that are not covered by a line in the table given in Step 6. Take the smallest element in this case the smallest element is 1. Subtract this smallest element from the uncovered cells and add 1 to elements (C1 and D1) that lie at the intersection of two lines. Finally, we get the new revised cost table, which is shown below:

	Jobs			
	1	2	3	4
A	0	4	0	6

Persons	B	0	2	6	0
	C	3	0	3	6
	D	3	0	0	0

Step 8:

Now, go to Step 4 and repeat the procedure until we arrive at an optimal solution (assignment).

Step 9:

Determine an assignment

Examine each of the four rows in the table given in Step 7, we may find that it is only row C which has only one zero box this cell C2 and cross out D2.

Note that all the remaining rows and columns have two zeros. Choose a zero arbitrarily, say A1 and box this cell so that the cells A3 and B1 get eliminated.

Now row B (cell B4) and column 3 (cell D4) has one zero box these cells so that cell D4 is eliminated.

Thus, all the zeros are either boxed or eliminated. This is shown in the following table

		Jobs			
		1	**2**	**3**	**4**
	A	[0]	4	~~0~~	6
Persons	**B**	~~0~~	2	6	[0]
	C	3	[0]	3	6

D	3	~~0~~	[0]	~~0~~

Since the number of assignments equal to the number of rows (columns), the assignment shown in the above tale is optimal.

The total cost of assignment is: 78 that is A1 + B4 + C2 + D3

20+17+17+24=78

QUESTION FOUR

Marking guide	**Marks**
a) Properties of a Poisson distribution in probability (1 Mark for each correct property)	**3**
b) (i) 1 Mark for elaborating data and 1 mark computing the probability	2
(ii) 0.5 for P (not playing football) 0.5 for p not playing handball and 1 mark for playing football or handball	2
Maximum	**4**
c) (i) 1 Mark for calculations and 1 Mark for correct answer	2
(ii) 1 Mark for calculations and 1 Mark for correct answer	2
Maximum	**4**
d) (i) 1 Mark to find A and 1 Mark to compute the probability	2
(ii) 1 Mark to find B and 1 Mark to compute the probability	2
(iii) 1 Mark to find C and 1 Mark to compute the probability	2
Maximum	**6**
e) Calculation of mean	1
Calculation of variance	1
Calculation of standard deviation	1
Maximum	**3**
Total Marks	**20**

Model Answers

a) The Poisson distribution is applicable in events that have a large number of rare and independent possible events. The following are the properties of the Poisson Distribution. In the Poisson distribution,

- The events are independent.
- The average number of successes in the given period of time alone can occur. No two events can occur at the same time.
- The Poisson distribution is limited when the number of trials n is indefinitely large.
- mean = variance = λ
- $np = \lambda$ is finite, where λ is constant.
- The standard deviation is always equal to the square root of the mean μ.
- If the mean is large, then the Poisson distribution is approximately a normal distribution.

b) Let the probability of students playing football be P(F) and playing handball be P(H),

then $P(F) = \frac{60}{100} = \frac{3}{5}, P(H) = \frac{50}{100} = \frac{1}{2}$ and $P(F \cap H) = \frac{30}{100} = \frac{3}{10}$

i) Probability of student playing football or handball means

$$P(F \cup C) = P(F) + P(C) - P(F \cap C)$$

$$= \frac{3}{5} + \frac{1}{2} - \frac{3}{10} = \frac{4}{5}$$

Therefore,

$$P(F \cup C) = 80\%$$

ii) Probability of play neither sport

$$P(\text{not playing football}) = P(\bar{F}) = 1 - P(F)$$
$$= 1 - \frac{3}{5} = \frac{2}{5}$$

$$P(\text{not playing handball}) = P(\bar{H}) = 1 - P(H)$$
$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Probability of playing neither sport $= P(\bar{F}) \times P(\bar{C})$

$$= \frac{2}{5} \times \frac{1}{2} = \frac{1}{5} = 20\%$$

c) m = 0.4

i) The probability that a record selected at random will have three defects (X=3)

$$P(X = 3) = \frac{\boldsymbol{e^{-m}\, m^3}}{3!} = \frac{\boldsymbol{e^{-0.4}\, (0.4)^3}}{3!} = \frac{\boldsymbol{0.6703 \times 0.064}}{6} = 0.00715$$

ii) The probability that a record selected at random will have three no defects (X=0)

$$P(X = 0) = \frac{\boldsymbol{e^{-m}\, m^0}}{0!} = \frac{\boldsymbol{e^{-0.4}\, (0.4)^0}}{1} = \frac{\boldsymbol{0.6703 \times 1}}{1} = 0.6703$$

d) Here the total number of wages earned are

N = 9+108+488+230+112+30+16+7 = 1,000.

i) Let A = number of wage earners having wages below 40

A = 9+108 = 117

$$P(A) = \frac{A}{\text{N}} = \frac{117}{1{,}000}$$

ii) Let B= number of wage earners having wages 55 or over
B = 30+16+7 = 53

$$P(B) = \frac{\text{B}}{\text{N}} = \frac{53}{1{,}000}$$

iii)Let C = number of wage earners with wages between 35-40 or 45-50
C = 108 + 230 = 338

$$P(C) = \frac{\text{C}}{\text{N}} = \frac{338}{1{,}000}$$

e) The Mean and Variance

- The mean of X is

$$\mu_X = E\left(X\right) = \sum_{k=1}^{5} x_k \times p_X(x_k)$$
$$= \left(-2\times\frac{1}{8}\right) + \left(-1\times\frac{1}{4}\right) + \left(0\times\frac{1}{5}\right) + \left(1\times\frac{1}{8}\right) + \left(2\times\frac{3}{10}\right) = \frac{9}{40} = 0.225.$$

- The variance of X is defined as $\sigma_X^2 = Var\left(X\right) = E\left[\left(X-\mu_X\right)^2\right] = \sum_{k=1}^{5}\left(x_k-\mu_X\right)^2 \times p_X(x_k).$

Using the formula $Var\left(X\right) = E\left(X^2\right) - \left(\mu_X\right)^2$ we obtain

$$Var\left(X\right) = \sum_{k=1}^{5} x_k^2 \times p_X(x_k) - \left(\mu_X\right)^2$$
$$= \left(4\times\frac{1}{8}\right) + \left(1\times\frac{1}{4}\right) + \left(0\times\frac{1}{5}\right) + \left(1\times\frac{1}{8}\right) + \left(4\times\frac{3}{10}\right) - \left(\frac{9}{40}\right)^2 = \frac{83}{40} - \frac{81}{1600} = \frac{3239}{1600} = 2.02.$$

The standard deviation is $\sigma_X = \sqrt{Var(X)} = \sqrt{2.02} \cong 1.42.$

QUESTION FIVE

Marking guide	Marks
a) 1 Mark if the candidate mentions "high volumes of sales are associated with high advertising costs"	1
1 Mark if says "low volumes of sales are associated with low advertising costs"	1
1 Mark if candidate says "the relationship between the two variables is a linear relationship"	1
Maximum	**3**
b) 1 Mark for maximin row and minimax column,	1
1 Mark for the 3x3 reduced matrix,	1
1 Mark for the 3x2 reduced matrix,	1
1 Mark for the 2x2 reduced matrix,	1
1 mark for p and 1-p,	1
1 Mark for r and 1-r,	1
1 Mark for v	1
1 Mark for the interpretation	1
Maximum	**8**
c) i) 1Mark for each correct column 4 and 5 (2 marks),	2
1 Mark for mean x and mean y,	1
one Mark for correct values of a and b	1
1 Mark for the equation of regression line	1
Maximum	**5**
ii) 1 Mark for x and y axes,	1
1 Mark for data	1
1 Mark for the trend line	1
Maximum	**3**
iii) 1 Mark for plugin and correct answer	**1**
Total Marks	**20**

Model Answers

(a) Looking at the above diagram we can see that high volumes of sales are associated with high advertising costs and that low volumes of sales are associated with low advertising costs. In other words, a relationship exists between the two variables, with the volume of sales increasing as the advertising cost goes up. As this increase is linear (i.e. the value of Y increases with the value of X in a linear way), the relationship between the two variables is *a linear* relationship

(b)First consider the minimum of each row.

Row	MinimumValue
1	2
2	3
3	3

Maximum of {2, 3, 3} = 3

Next consider the maximum of each column

Column	Maximum Value
1	6
2	4
3	4
4	7
4	6

Minimum of {6, 4, 7, 6}= 4

It is clear that Max {row minima} □ min {column maxima}. Therefore, we see that there is no saddle point for the game underconsideration.

Compare the columns II and III

Column II	Column III
2	3
4	7
3	5

We see that each element in column III is greater than the corresponding element in column II. The choice is for player B. Since column II dominates column III, player B will discard his strategy 3. Now we have the reduced game

$$\begin{array}{cc} & \begin{array}{ccc} I & II & IV \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 4 & 2 & 6 \\ 3 & 4 & 5 \\ 6 & 3 & 4 \end{bmatrix} \end{array}$$

For this matrix again, there is no saddle point. Column II dominatescolumn IV. The choice is for player B. So player B will give up his strategy 4. The game reduces to the following:

$$\begin{array}{cc} & \begin{array}{cc} \text{I} & \text{II} \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 4 & 2 \\ 3 & 4 \\ 6 & 3 \end{bmatrix} \end{array}$$

This matrix has no saddle point too.

The third row dominates the first row. The choice is for player A.He will give up his strategy 1 and retain strategy 3. The game reduces to the following:

$$\begin{bmatrix} 3 & 4 \\ 6 & 3 \end{bmatrix}$$

Again, there is no saddle point. We have a 2x2 matrix. Take this matrix as:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Then we have a = 3, b = 4, c = 6 and d = 3. Use the formulae for p, 1-p, r,1-r and V.

$p = \frac{d-c}{(a+d)-(b+c)} = \frac{3-6}{(3+3)-(6+4)} = \frac{-3}{6-10} = \frac{-3}{-4} = \frac{3}{4},$

$$1 - p = 1 - \frac{3}{4} = \frac{1}{4},$$

$$r = \frac{d-b}{(a+d)-(b+c)} = \frac{3-4}{(3+3)-(6+4)} = \frac{-1}{6-10} = \qquad = \frac{-1}{-4} = \frac{1}{4},$$

$$1 - r = 1 - \frac{1}{4} = \frac{3}{4}$$

The value of game is now:

$$v = \frac{ad - bc}{(a+d) - (b+c)} = \frac{3 \times 3 - 4 \times 6}{-4} = \frac{-15}{-4} = \frac{15}{4}$$

Thus, $X = (\frac{3}{4}, \frac{1}{4}, 0,0)$ and $Y = (\frac{1}{4}, \frac{3}{4}, 0,0)$ are the optimal strategies.

(c)(i) Using normal equations and the sugar production data we can compute constants a and b as shown in Table 1:

Table 1: Calculations for Least Squares Equations

Year	Time Period (x)	Production (x)	x^2	xy	Trend Values $\bar{y}$
1992	1	80	1	80	84
1993	2	90	4	180	86
1994	3	92	9	276	88
1995	4	83	16	336	90
1996	5	94	25	470	92
1997	6	99	36	594	94
1998	7	92	49	644	96
Total	**28**	**630**	**140**	**2576**	

$$\bar{x} = \frac{\sum x}{n} = \frac{28}{7} = 4,$$

$$\bar{y} = \frac{\sum y}{n} = \frac{630}{7} = 90$$

$$b = \frac{\sum xy - n\overline{xy}}{\sum x^2 - n(\bar{x})^2} = \frac{2576 - 7(4)(90)}{140 - 7(4)^2} = \frac{56}{28} = 2$$

$$a = \bar{y} - b\bar{x} = 90 - 2(4) = 82$$

Therefore, linear trend component for the production of sugar is:

$$\hat{y} = a + bx = 82 + 2x$$

The slope $b = 2$ indicates that over the past 7 years, the production of sugar had an average growth of about 2 thousand quintals per year.

(ii) Plotting points on the graph paper, we get an actual graph representing production of sugar over the past 7 years. Join the point $a = 82$ and $a = 2$ (corresponds to 1993) on the graph we get a trend line as shown in Fig.1.

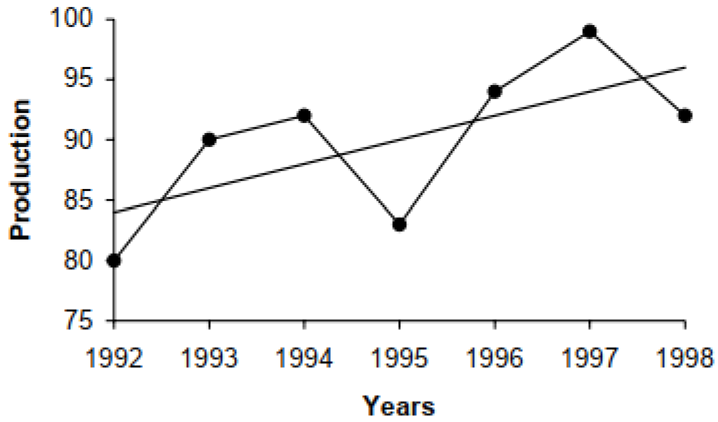


Figure 1: Linear Trend of Production of Sugar

(iii) The production of sugar for year 2001 will be $\hat{y} = 82 + 2(10) = 102$ thousand quintals.

QUESTION SIX

Marking guide	Marks
a) Advantages of Arithmetic mean (1 Mark for each correct advantage)	**2**
b) i) 1 Mark for mean Rubavu and mean Karongi,	1
1 Mark for variance Rubavu,	1
1 Mark for variance Karongi,	1
1 Mark for std Rubavu	1
1 Mark for std Karongi	1
Maximum	**5**
ii) 1 Mark for C.V Rubavu	1
1 Mark for C.V Karongi	1
Maximum	**2**
iii) 2 marks for interpretation	**2**
c) 1 Mark for mean, 1 Mark for mode and 1 Mark for the coefficient	**3**
d) 1 Mark for expected NPV of A and B (1 Mark for column 3 and 5 for both table A and B proposals (4 Marks), 1 Mark for both SA and SB)	**6**
Total Marks	**20**

Model Answers

(a) Advantages of Arithmetic mean:

- Arithmetic mean rigidly defined by Algebraic Formula
- It is easy to calculate and simple to understand
- It is based on all observations of the given data.
- It is capable of being treated mathematically hence it is widely used in statistical analysis.
- Arithmetic mean can be computed even if the derailed distribution is not known but some of the observation and number of the observation are known.
- It is least affected by the fluctuation of sampling.
- For every kind of data mean can be calculated.

(b)Results are as follows R stands for Rubavu, whereas K stands for Karongi

LB	UB	X	fR	fK	$fR \times X$	$fK \times X$	X^2	$fR \times X^2$	$fK \times X^2$
24	28	26	8	19	208	494	676	5408	12844
28	32	30	41	36	1230	1080	900	36900	32400
32	36	34	77	47	2618	1598	1156	89012	54332
36	40	38	90	58	3420	2204	1444	129960	83752
40	44	42	58	27	2436	1134	1764	102312	47628
44	48	46	26	13	1196	598	2116	55016	27508
Total			**300**	**200**	**11108**	**7108**		**418608**	**258464**

Rubavu mean = $\frac{11108}{300} = 37$

Karongi mean = $\frac{7108}{200} = 35.5$

Rubavu Variance = $\frac{418608}{300} - 37^2 = 24.4$

Karongi Variance =$\frac{258464}{200} - 35.5^2 = 29.2$

Rubavu std = 4.9

Karongi std = 5.4

ii) Rubavu C.V = 4.9/37x100=13.3%

Karongi C.V = 5.4/35.5x100 = 15.2%

iii)From results got in (a) and (b), it is clear that the relative dispersion between the distribution is not wide. This calculation is more appropriate where there is substantial difference between the means of the distribution. The coefficient is a measure to quantify the relative dispersion. Karongi has the higher in both mean and C.V than Rubavu.

(c)

Height	**f**	**X**	**d=X-67 A=67**
60-62	5	61	-6
63-65	18	64	-3
66-68	42	67	0
69-71	27	70	3
72-74	8	73	6
Total	100		

i)

$$Mean = A + \frac{\sum fd'}{N} \times i = 67 + \frac{15}{100} \times 3$$

$$= \ 67 + \frac{45}{100} = 67 + 0.45 = \qquad =$$

67.45

Mode lies in the class 66-68 as maximum frequency occurs in this class. Using the formula,

$$Mode = L_1 + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times i$$

$$=66 + \frac{42-18}{2\times42-18-27} \times 3 = 66 + 1.846 =$$

$$= \ 67.846$$

Coefficient of Skewness = $\frac{Mean-Mode}{S.D} = \frac{|67.45-67.846|}{2.919} = 0.13$ (1 marks)

(d)The expected (average) net present value for both the proposals is:

Proposal A: Expected NPV $= 1{,}559 \times 0.30 + 5{,}662 \times 0.40 + 9{,}175 \times 0.30$

= 467.7+2,264.8+2,752.5

= 5,485

Proposal B: Expected NPV $= -10{,}050 \times 0.30 + 5{,}812 \times 0.40 + 20{,}584 \times 0.30$

= -3,015+2,324.8+6,175.2

= 5,485

Since the expected NPV in both projects is the same, Mr. Nsabimana would like to choose less risky proposal. For this, we have to calculate the standard deviation in both the cases.

Table: Standard deviation for proposal A:

NPV (x_i)	Expected value NPV ($\bar{x}$)	$x - \bar{x}$	Probability of NPV (f)	$f(x - \bar{x})^2$
1,559	5,485	-3,926	0.30	4,624,042.8

5,662	5,485	177	0.40	12,531.6
9,175	5,485	3,690	0.30	4,084,830
			1,00	8,721,404.4

$$s_A = \sqrt{\frac{\sum f(x-\bar{x})^2}{\text{N}}} = \sqrt{8,721,404.4} = 2,953.20$$

Table: Standard deviation for proposal B:

NPV (x_i)	Expected value NPV ($\bar{x}$)	$x - \bar{x}$	Probability of NPV (f)	$f(x - \bar{x})^2$
-10,050	5,485	-15,535	0.30	72,400,867.5
5,812	5,485	327	0.40	4,2771.6
20,584	5,485	15,099	0.30	68,393,940
			1,00	140,837,579

$$s_B = \sqrt{\frac{\sum f(x-\bar{x})^2}{\text{N}}} = \sqrt{140,837,579} = 11,867.50$$

Advise Mr. Nsabimana to choose the proposal A since it has the least standard deviation.

QUESTION SEVEN

Marking guide	**Marks**
a) 1 Mark for Total revenue,	1
1 Mark for marginal revenue,	1
1 Mark for Marginal Cost,	1
1 Mark for computing the value of q	1
1 Mark for interpretation	1
Maximum	**5**
b) 3 Marks upon getting a correct answer after the multiplication	**3**
c) 1 Mark after substituting the matrices and 1 Mark once the final answer is correct	**2**
d) i) 1 Mark computing BEP, 0.5 Mark computing the total cost, 1 Mark for drawing the sales revenue line, 1 Mark for total cost line, 0.5 Mark for fixed cost line, 1 Mark for axes	**5**
ii) 1 Mark for BEP	**1**
iii) 1 Mark for quantity to produce to get profit	**1**
iv) 1 Mark to compute the profit	**1**
v) 1 Mark to compute the marginal safety and 1 Mark for the percentage	**2**
Total Marks	**20**

Model Answers

(a)Given Total Revenue $(TR) = pq = (184 - 4q)q = 184q - 4q^2$, then the Marginal Revenue

$$MR = \frac{\text{dTR}}{\text{dq}} = 184 - 8q$$

The Marginal Cost (MC) = $\frac{\text{dTC}}{\text{dq}} = 3q^3 - 42q + 160$

To maximize profits MC = MR. Therefore,

$$3q^2 - 42q + 160 = 184 - 8q$$

$$3q^2 - 34q - 24 = 0$$

$$(q - 12)(3q + 2) = 0 \Rightarrow q - 12 = 0 \; or \; 3q + 2 = 0$$

It gives that $q = 12 \; or \; q = -\frac{2}{3}$

One cannot produce a negative quantity and so the firm must produce 12 units of output in order to maximize profits

(b)The demand oil is calculated as

$$q = \beta x = (4.2 \;\; -0.1 \;\; 0.4 \;\; 0.2 \;\; -0.1 \;\; 0.2)\begin{pmatrix} 1 \\ 30 \\ 18.5 \\ 52 \\ 12.8 \\ 61 \end{pmatrix} = 29.92$$

(c)Total sales for each week will simply be the sum of the corresponding elements in matrices **A** and **B**. For example, in week 1 the total sales of product Q will be 5 plus 8. Total combined sales for Q and R can therefore be represented by the matrix.

$$T = A + B = \begin{pmatrix} 5 & 4 & 12 & 7 \\ 10 & 12 & 9 & 14 \end{pmatrix} + \begin{pmatrix} 8 & 9 & 3 & 4 \\ 8 & 18 & 21 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 5+8 & 4+9 & 12+3 & 7+4 \\ 10+8 & 12+18 & 9+21 & 14+5 \end{pmatrix} = \begin{pmatrix} 13 & 13 & 15 & 11 \\ 18 & 30 & 30 & 19 \end{pmatrix}$$

d) Fixed cost (FC) = 80,000

Variable cost per unit (VC) =4

Selling price of each item (P) = 20

Estimated sales = 200,0000

Hence, number of units sold = $\frac{200,000}{20} = 10,000$

Purchase/selling price for total unit (P) = 20 x 10,000 =200,000

For $BEP = \frac{F}{1-\frac{V}{P}} = \frac{80,000}{1-\frac{4\times10,000}{2\times10,000}} = 100,000$

Variable Cost = No of units x variable cost per unit

$$V.C = 10,000 \times 4 = FRW\ 40,000$$

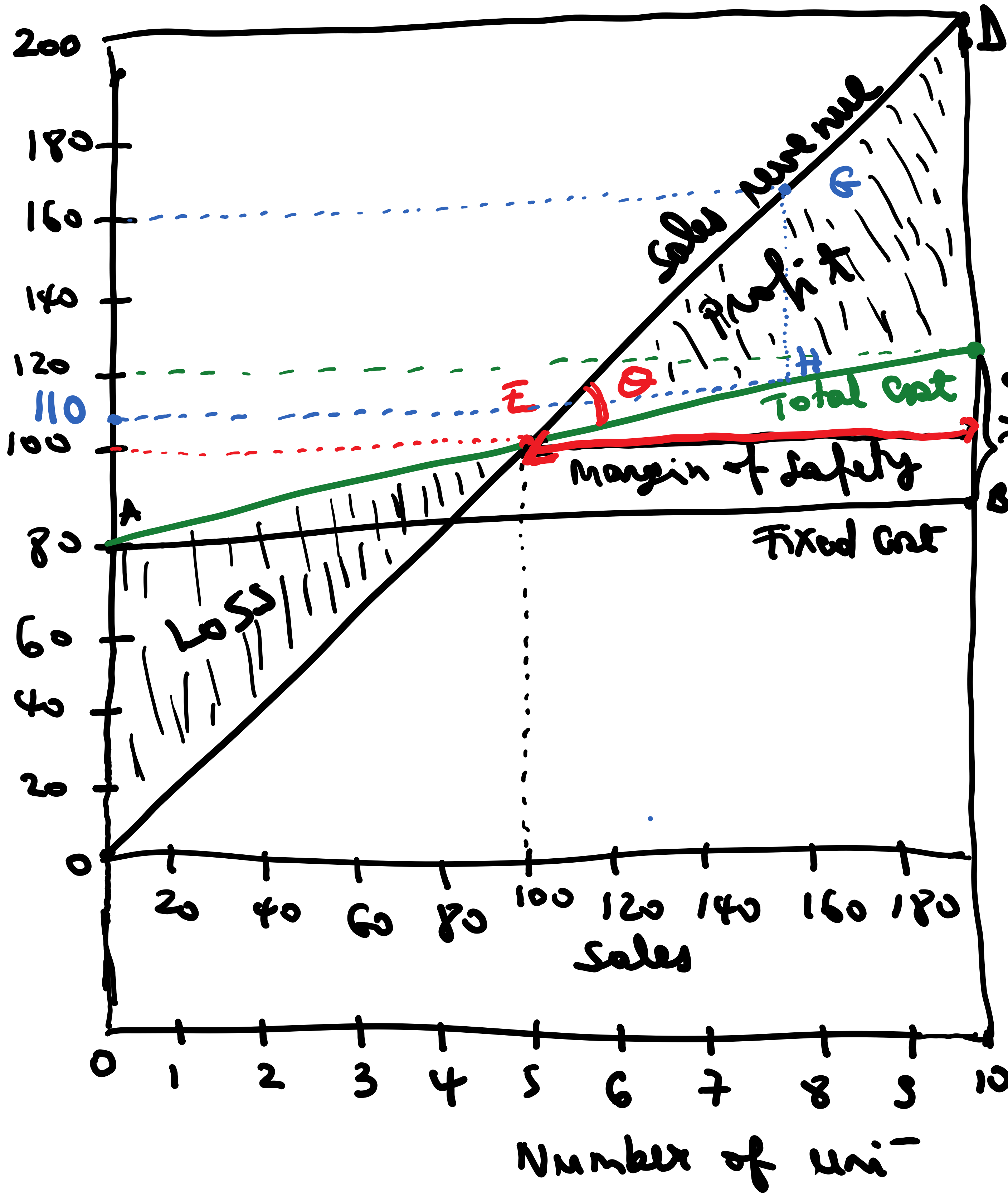
Total Cost = Fixed Cost + Variable cost

= 80,000 + 40,000 = FRW 120,000

(i) Procedure to draw BE chart:

- Draw the fixed cost line(AB) at FRW 80000 on the graph paper.
- Variable Cost varies from 0 at 0 unit to FRW 40000 at 10000 units.
- Draw variable cost line (AC) above the fixed cost line. The variable cost when added to fixed cost gives the total cost of FRW 120,000.
- Sales revenue is zero at 0 units and it is 200,000 at 10,000 units. Therefore, draw the sales revenue line OD.

The following is the break-even chart


200
180
160
140
120
110
100
80
60
40
20
0
A
B
C
D
E
G
H
θ
Sales Revenue
Profit
Total Cost
Margin of Safety
Fixed Cost
Loss
20
40
60
80
100
120
140
160
180
Sales
0
1
2
3
4
5
6
7
8
9
10
Number of uni

(ii) In the break-even chart, point E represents the break-even point. It is at 5000 units or FRW100,000, i.e., where production when sold will return FRW100,000 in reverse to the company.

(iii) The company should produce and sell more than 5000 to seek profit.

(iv) To determine the profit earned (PE) at a turnover of FRW160,000, from the figure,

PE= 160,000-110,00 = FRW 50,000

(v)The margin of safety (M.S) at 10,000 units = FRW 200,000 – FRW 100,000 = FRW 100,000 corresponding to 50 % of total sales.

END OF MARKING GUIDE AND MODEL ANSWERS